Corrections to: *Codimension 2 cycles on Severi-Brauer varieties and decomposability*

In Example 4.3 it's claimed that for any odd prime p and for any integers $0 < b \leq a$ one can find central simple k-algebras $D_0, D_1, ..., D_{a-b}$ over the field $k = \mathbb{Q}$ satisfying the following four properties:

(1) $\mathrm{ind}(D_0) = \exp(D_0) = p^b$
(2) $\mathrm{ind}(D_i) = \exp(D_i) = p$ for all $i > 1$
(3) $\mathrm{ind}(D_0 \otimes D_1 \otimes \cdots \otimes D_{a-b}) = p^a$
(4) $\exp(D_0 \otimes D_1 \otimes \cdots \otimes D_{a-b}) = p^b$.

This claim is then used, in both the abstract and in Example 4.3, to show that there exists a central division $\mathbb{Q}$-algebra A of index n and exponent m, for any pair of positive odd integers (n, m) with m dividing n and with m and n sharing the same prime factors, such that each of the primary algebra factors of A admits a decomposition like the above. However, over $\mathbb{Q}$, every division algebra has equal index and exponent [Rei03, Theorem 32.19] so that (3) and (4) can not both hold if $a \neq b$.

Examples of algebras $D_0, D_1, ..., D_{a-b}$ satisfying the properties (1) - (4) above do exist over extensions of $\mathbb{Q}$, however. To see this, let $k = \mathbb{Q}(\zeta_{p^b})(x_0, y_0, ..., x_{a-b}, y_{a-b})$ where ζ_{p^b} is a primitive p^bth root of unity. For any $1 \leq j \leq b$, let $(x_i, y_i)_{p^j}$ be the symbol algebra over k with generators $1, u_i, v_i$ and relations

$$u_i^{p^j} = x_i, \quad v_i^{p^j} = y_i, \quad \text{and} \quad u_i v_i = \zeta_{p^b}^{p^{b-j}} v_i u_i.$$

Then, the algebras

$$D_0 = (x_0, y_0)_{p^b},\ D_1 = (x_1, y_1)_p,\ ...,\ D_{a-b} = (x_{a-b}, y_{a-b})_p$$

have the specified properties (1) - (4) by [TW87, Example 3.6 and Theorem 4.7 (i)].

For the general case, let (n, m) be a pair of positive odd integers with m dividing n and with m and n having the same prime factors. Let

$$m = p_1^{b_1} \cdots p_r^{b_r} \quad \text{and} \quad n = p_1^{a_1} \cdots p_r^{a_r}$$

be primary factorizations of m, n so that p_i is prime and $0 < b_i \leq a_i$ for each $1 \leq i \leq r$. For each $1 \leq i \leq r$, let $\xi_i = \zeta_{p^{b_i}}$ be a p^{b_i}th primitive root of unity and let ζ_m be an mth primitive root of unity. Consider the transcendental field extension of $\mathbb{Q}(\zeta_m)$,

$$k = \mathbb{Q}(\zeta_m)(x_0^{(1)}, y_0^{(1)}, ..., x_{a_1-b_1}^{(1)}, y_{a_1-b_1}^{(1)}, ..., x_0^{(r)}, y_0^{(r)}, ..., x_{a_r-b_r}^{(r)}, y_{a_r-b_r}^{(r)}).$$

For any $1 \leq i \leq r$, for any $0 \leq j \leq a_i - b_i$, and for any $1 \leq l \leq b_i$, let $(x_j^{(i)}, y_j^{(i)})_{p_i^l}$ be the symbol algebra with generators $1, u_j, v_j$ and relations

$$u_j^{p_i^l} = x_j^{(i)}, \quad v_j^{p_i^l} = y_j^{(i)}, \quad \text{and} \quad u_j v_j = \xi_i^{p_i^{b_i-l}} v_j u_j.$$

Finally, the algebra

$$A = \bigotimes_{i=1}^{r} A_i \quad \text{where} \quad A_i = (x_0^{(i)}, y_0^{(i)})_{p_i^{b_i}} \otimes \cdots \otimes (x_{a_i-b_i}^{(i)}, y_{a_i-b_i}^{(i)})_{p_i}$$

has the desired properties, again by [TW87, Example 3.6 and Theorem 4.7 (i)].

References

[Rei03] I. Reiner, *Maximal orders*, London Mathematical Society Monographs. New Series, vol. 28, The Clarendon Press, Oxford University Press, Oxford, 2003, Corrected reprint of the 1975 original, With a foreword by M. J. Taylor. MR 1972204

[TW87] J.-P. Tignol and A. R. Wadsworth, *Totally ramified valuations on finite-dimensional division algebras*, Trans. Amer. Math. Soc. **302** (1987), no. 1, 223–250. MR 887507